Asymptotic Representation of a Two-Dimensional Robotic Mechanism around a Special Case

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A B S T R A C T

The connection equation of the studied mechanism is expressed in the form of the following system of nonlinear algebraic equations:[1]

A R T I C L E I N F O

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$$F_i(U,V) \stackrel{\text{def}}{=} F_i(x_1,\dots,x_m,x_{m+1},\dots,x_{m+n}) = 0,\;\; i = \overline{1,m}\;\; n \le m \qquad (1)$$

In this, $U = (x_1, x_2, \dots, x_m)$ and $V = (x_{m+1}, \dots, x_{m+n})$ state and control coordinates as appropriate, F_i- and polynomials. The number of equations m, state corresponds to the number of coordinates U, and n is called the degree of freedom of the mechanism. In general, the degree of freedom of the mechanism is found by the equation *n=k-m* [1], where k is the number of unknowns in the system of equations. The values of each control coordinate *V* correspond to a finite number of values of *U* that satisfy the system (1). *U(V)* is called a multivalued vector function, i.e $x_i = x_i(x_{m+1}, \dots, x_{m+n})$[2-3]:

Let's assume, $x_k^0 = (x_1, \dots, x_m, x_{m+1}, \dots, x_{m+n})$ let point (1) satisfy the system of equations. Let us observe the position of the state function near this point. For this, we look at the Jacobian matrix of the $m \times k$-dimensional system (1):

$$J \stackrel{\text{def}}{=} \left(\frac{\partial F_i}{\partial x_k}\right), i = \overline{1,m}, k = \overline{1,m+n}. \qquad (2)$$

$1 \le i_1 < i_2 <..< i_m \le m+n$ let *m* be *a* group of ordered integers, we denote it by $I=(i_1,...i_m)$. By M_I, we denote the minors of order m of the matrix (2), that is, it is obtained from the selection of columns with indices $i_1,...i_m$.

Let's consider the following situation [1]:

1) If $\forall\, I,\; M_I(x_k^0) \ne 0$ if it is fulfilled, then we call the point x_k^0 (1) a simple point of the system;
2) If $\exists\, I', I'', M_{I'}(x_k^0) = 0, M_{I''}(x_k^0) \ne 0$ is fulfilled, then the point x_k^0 is (1)
3) we call it a special point of the first type of the system of equations.
4) If $\forall\, I,\; M_I(x_k^0) = 0$ is fulfilled, then we call the point x_k^0 a special point of the second type of the system of equations (1).

The solution of the system (1) around a simple point can be described in the form of absolutely converging series based on the Cauchy theorem about non-expandable functions:

$$x_i - x_i^0 = \varphi_i(x_{m+j} - x_{m+j}^0), \quad i = \overline{1,m}, j = \overline{1,n}, \qquad (3)$$

In this, φ_i-graded rows. Accordingly, the state of the mechanism is simple at this point. The first and second type correspond to the special cases of the mechanism around the special points. Around a special point, the state function *U(V)* ceases to be a one-valued analytic function of control coordinates *V*. The solution of the system (1) around the second type of special point (dead state) cannot be expressed in the form (3), because in this case the conditions of the theorem about non-revealing functions are not fulfilled, but using the method of Newton's polynomials, parametric solutions of the following form can be obtained can be obtained [2-3]:

$$x_i = x_i^0 + \sum_{j=1}^{\infty} b_{ij}\tau^{p_{ij}}, i = 1, m \qquad (4)$$

It follows from the above that (1) To analyze the system solutions, it is necessary to find all the special points and show all the cases where the second type of special points are. Based on the analysis of the peculiarities of the connection equations of the studied mechanisms, the algorithm for calculating the following peculiarities was developed (see [1]):

1. Write the connection equations of the mechanism in algebraic form;

$$F_i(U,V) = 0, i = 1,2, \dots, m.$$

2. Compilation of the Jacobian matrix of the system of connection equations;

$$J \stackrel{\text{def}}{=} \left(\frac{\partial F_i}{\partial x_j}\right), \quad i = \overline{1,m}, \quad j = 1, m+n$$

3. J select all m-order M_I minors of the Jacobian matrix;

4. Calculation of selected minors;

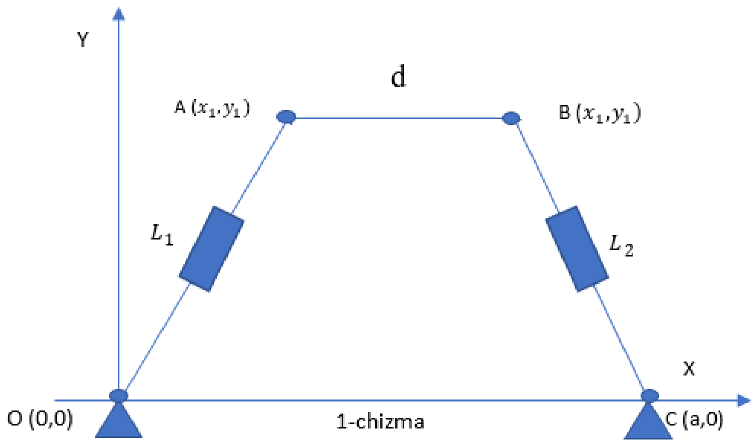
5. It is necessary to divide the calculated minors into multipliers and determine the state of the mechanism in which the condition $M_{I'}=0$ is fulfilled for at least one index group I';

6. Finding the conditions for the simultaneous zeroing of all m-order minors of the Jacobi matrix;

7. Identify the states of the mechanism where the condition $M_I=0$ is fulfilled for all $I = (i_1, \dots, i_m)$.

The first 5 steps are enough to find the first type of features, the last two steps, i.e. 6 and 7, are very difficult to determine.

EXAMPLE. Let us consider the asymptotic solution of the rectangular hydrocylindrical mechanism around a special point. Here, points A, O, B, C have the following coordinates: $O(0,0)$, $A(x_1, y_1), B(x_2, y_2), C(a, 0)$ and $AB = d$ positive magnitude, L_1, L_2 positive variable quantity.



The connection equations of the state functions of the considered mechanism have the following form:

$$\begin{cases} f_1 \stackrel{\text{def}}{=} x_1^2 + y_1^2 = L_1^2 \\ f_2 \stackrel{\text{def}}{=} (x_2 - x_1)^2 + (y_2 - y_1)^2 = d_2^2 \\ f_3 \stackrel{\text{def}}{=} (a - x_2)^2 + y_2^2 = L_2^2 \end{cases} \tag{5}$$

The system (1) consists of 3 equations and 6 unknowns, from which it can be seen that it has *n=3* degrees of freedom, that is, it consists of a mechanism with 3 degrees of freedom. The 3x6 Jacobian matrix of system (1) looks like this will be

$$J = 2\begin{pmatrix} x_1 & y_1 & 0 & 0 & 0 & -L_1 \\ x_2 - x_1 & y_1 - y_2 & x_2 - x_1 & y_2 - y_1 & 0 & 0 \\ 0 & 0 & x_2 - a & y_2 & 0 & -L_2 \end{pmatrix}$$

The 3rd order minors of the matrix are as follows:

$$M_1 = 8\begin{vmatrix} x_1 & y_1 & 0 \\ x_2 - x_1 & y_1 - y_2 & x_2 - x_1 \\ 0 & 0 & x_2 - a \end{vmatrix} \qquad M_2 = 8\begin{vmatrix} x_1 & y_1 & 0 \\ x_2 - x_1 & y_1 - y_2 & y_2 - y_1 \\ 0 & 0 & y_2 \end{vmatrix}$$

$$M_3 = 8\begin{vmatrix} x_1 & y_1 & 0 \\ x_2 - x_1 & y_1 - y_2 & 0 \\ 0 & 0 & 0 \end{vmatrix} \qquad M_4 = 8\begin{vmatrix} x_1 & 0 & -L_1 \\ x_2 - x_1 & y_2 - y_1 & 0 \\ 0 & 0 & -L_2 \end{vmatrix}$$

$$M_5 = 8\begin{vmatrix} x_1 & 0 & 0 \\ x_2 - x_1 & x_2 - x_1 & y_1 - y_2 \\ 0 & x_2 - a & y_2 \end{vmatrix} \qquad M_6 = 8\begin{vmatrix} x_1 & 0 & 0 \\ x_2 - x_1 & x_2 - x_1 & 0 \\ 0 & x_2 - a & 0 \end{vmatrix}$$

$$M_7 = 8\begin{vmatrix} x_1 & 0 & -L_1 \\ x_2 - x_1 & x_2 - x_1 & 0 \\ 0 & x_2 - a & -L_2 \end{vmatrix} \qquad M_8 = 8\begin{vmatrix} x_1 & 0 & 0 \\ x_2 - x_1 & y_2 - y_1 & 0 \\ 0 & y_2 & 0 \end{vmatrix}$$

$$M_9 = 8\begin{vmatrix} x_1 & 0 & -L_1 \\ x_2 - x_1 & y_2 - y_1 & 0 \\ 0 & y_2 & -L_2 \end{vmatrix} \qquad M_{10} = 8\begin{vmatrix} x_1 & 0 & -L_1 \\ x_2 - x_1 & 0 & 0 \\ 0 & 0 & -L_2 \end{vmatrix}$$

$$M_{11} = 8\begin{vmatrix} y_1 & 0 & 0 \\ y_1 - y_2 & x_2 - x_1 & y_2 - y_1 \\ 0 & x_2 - a & y_2 \end{vmatrix} \qquad M_{12} = 8\begin{vmatrix} y_1 & 0 & 0 \\ y_1 - y_2 & x_2 - x_1 & 0 \\ 0 & x_2 - a & 0 \end{vmatrix}$$

$$M_{13} = 8\begin{vmatrix} y_1 & 0 & -L_1 \\ y_1 - y_2 & x_2 - x_1 & 0 \\ 0 & x_2 - a & -L_2 \end{vmatrix} \qquad M_{14} = 8\begin{vmatrix} y_1 & 0 & 0 \\ y_1 - y_2 & y_2 - y_1 & 0 \\ 0 & x_2 - a & 0 \end{vmatrix}$$

$$M_{15} = 8\begin{vmatrix} y_1 & 0 & -L_1 \\ y_1 - y_2 & 0 & 0 \\ 0 & 0 & -L_2 \end{vmatrix} \qquad M_{16} = 8\begin{vmatrix} 0 & 0 & 0 \\ x_2 - x_1 & y_2 - y_1 & 0 \\ x_2 - a & y_2 & 0 \end{vmatrix}$$

$$M_{17} = 8\begin{vmatrix} 0 & 0 & -L_1 \\ x_2 - x_1 & y_2 - y_1 & 0 \\ x_2 - a & y_2 & -L_2 \end{vmatrix} \qquad M_{18} = 8\begin{vmatrix} y_1 & 0 & -L_1 \\ y_2 - y_1 & x_2 - x_1 & 0 \\ 0 & x_2 - a & -L_2 \end{vmatrix}$$

$$M_1 = 8(x_2 - a)(x_1y_2 - x_2y_1); \quad M_2 = 8y_2(x_1y_2 - x_2y_1);$$

$$M_3 = 8L_1(x_2y_1 - x_1y_2)$$

$$M_4 = 8x_1y_1(x_2 - x_1);$$

$$M_5 = 8(x_2 - x_1)(L_2(a - x_2) - x_1L_2);$$

$$M_6 = 8(x_1L_2(y_1 - y_2) + x_1y_2L_1 - L_1y_2x_2x_1);$$

$$M_7 = 8y_1(y_2(a - x_1) - y_1(x_2 - a));$$

$$M_8 = 8(y_1(L_2(x_1 - x_2) - L_1(x_2 - a)) - L_1y_2(x_2 - a));$$

$$M_9 = 8L_1\left(y_2(x_1 - a) + y_1(a - x_2)\right);$$

$$M_{10} = 8(y_1(L_2(x_1 - x_2) + L_1(x_2 - a)) - L_1y_2(x_2 - a));$$

$$M_{11} = M_{12} = M_{13} = M_{14} = M_{15} = M_{16} = M_{17} = M_{18} = M_{19} = M_{20} = 0$$

$$M_{19} = 8\begin{vmatrix} 0 & 0 & -L_1 \\ x_2 - x_1 & 0 & 0 \\ x_2 - a & 0 & -L_2 \end{vmatrix} \qquad M_{20} = 8\begin{vmatrix} 0 & 0 & -L_1 \\ y_2 - y_1 & 0 & 0 \\ y_2 & 0 & -L_2 \end{vmatrix}$$

Theorem. The mechanism defined by the system of coupling equations (5) does not achieve the second type of specificity.

Proof. According to the condition, the condition $M_i = 0$ ($i = \overline{1,20}$) must be fulfilled for $\forall$ i for the mechanism to achieve the second type of specificity (Figure 3). But this situation does not happen, because according to the definition, $L_{1,}L_2, a$>0 Theorem is proved.

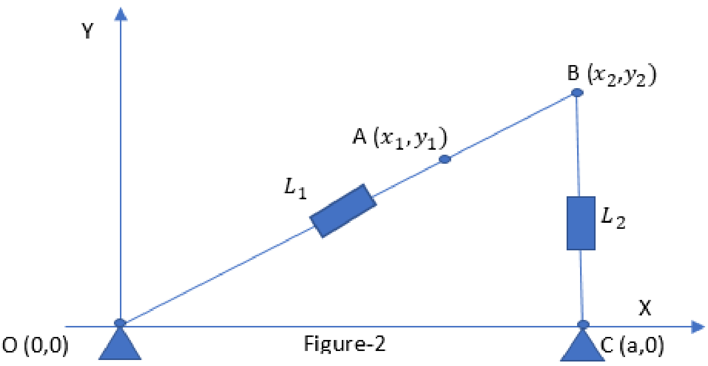


Figure-2

$x_2 = a;\ M_2 = 0$

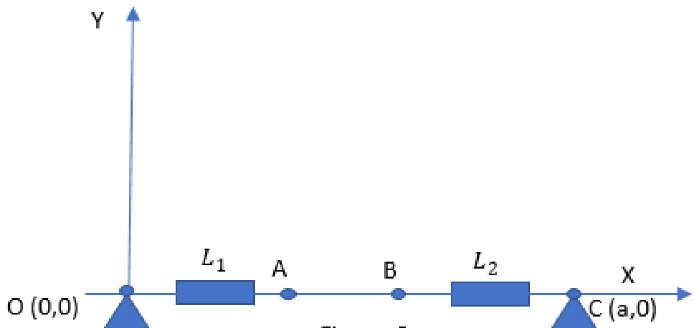


Figure-3

$$L_1^0 = x_1^0; \quad L_2^0 = a - x_2^0\,; \quad y_1^0 = y_2^0 = 0\,; \quad d = x_2^0 - x_1^0\,;$$

$$M_1 = M_2 = M_3 = M_4 = M_5 = M_6 = M_7 = M_8 = M_9 = M_{10} = M_{11} = M_{12} = M_{13} = M_{14} = M_{15} = M_{16} = M_{16} = M_{17} = M_{18} = M_{19} = M_{20} = 0$$

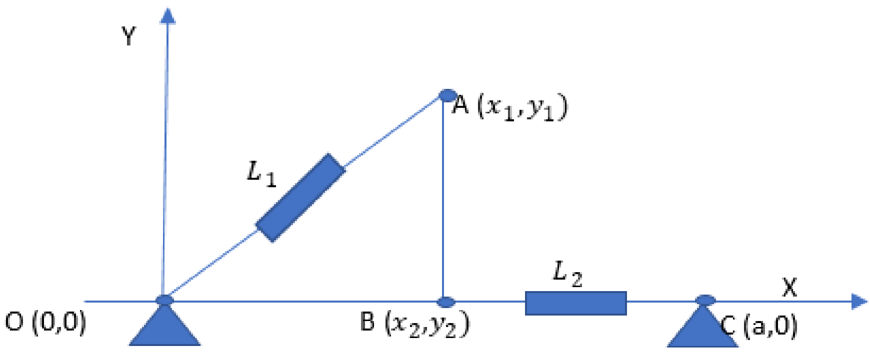


Figure-4

$$x_1 = x_2; \; M_7 = M_5 = 0$$

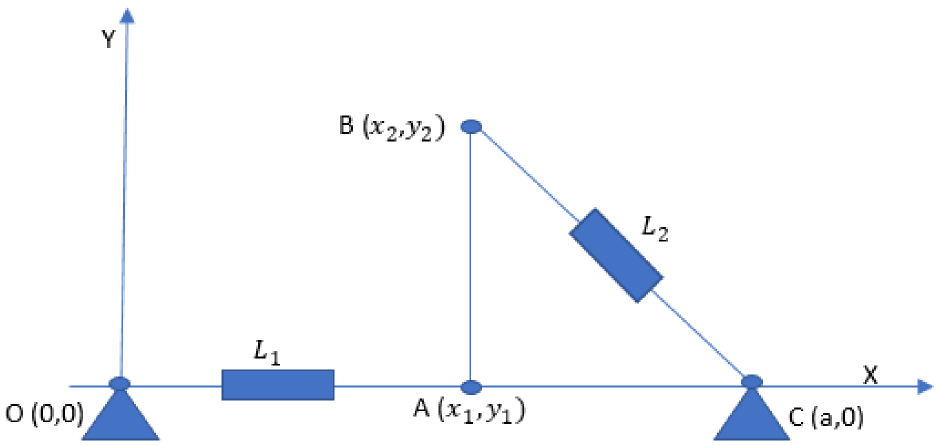
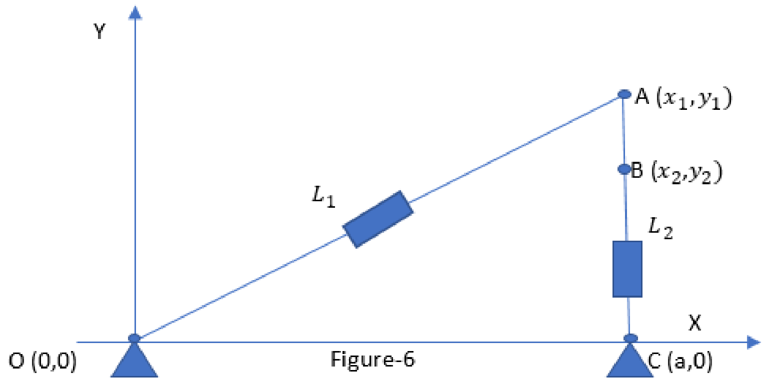


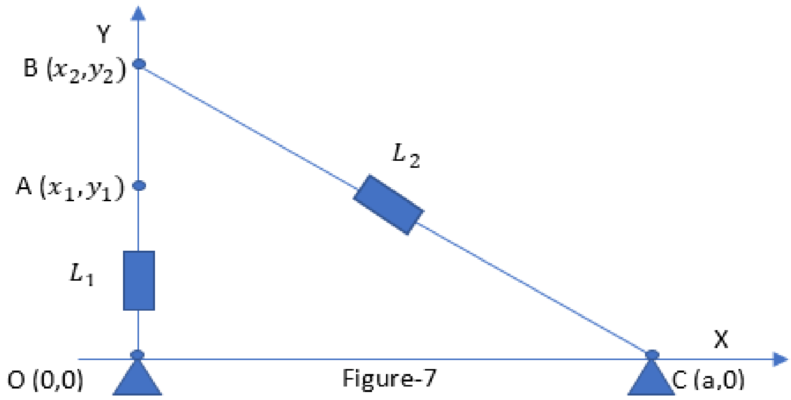
Figure-5

$$x_1 = x_2; \quad M_7 = 0$$



$$x_2 = x_2 = a;$$

$$M_5 = M_7 = M_{11} = M_{13} = M_{17} = M_{18} = 0$$



$$x_2 = x_2 = 0;$$

$$M_1 = M_2 = M_4 = M_7 = M_9 = 0$$

Now, using the method of Newton's polynomials, we look for parametric solutions of system (1) in a small neighborhood of the special point P^0. In this case, let's take the following coordinate change in the system $P^0(x_1^0, x_2^0, 0,0, L_{1,}^0 L_2^0)$(Figure 3.)

$$\begin{cases} x_1 = x_1^0 + z_1 \\ x_2 = x_2^0 + z_2 \\ y_1 = z_3 \\ y_2 = z_4 \\ L_1 = L_1^0 + z_5 \\ L_2 = L_2^0 + z_6 \end{cases} \qquad (6)$$

$$L_1^0 = x_1^0; \quad L_2^0 = a - x_2^0; \quad y_1^0 = y_2^0 = 0; \quad d = x_2^0 - x_1^0;$$

Here z_i $(i = \overline{1,6})$ is a small deviation from the special point P^0. Putting these values in system (1), we create the following system:

$$\begin{cases} \hat{f}_1 \triangleq (x_1^0 + z_1)^2 + z_3^2 = (x_1 + z_5)^2 \\ \hat{f}_2 \triangleq (x_1^0 + z_1 - x_2^0 - z_2)^2 + (z_3 - z_4)^2 = (x_2 - x_1)^2 \\ \hat{f}_3 \triangleq (x_2^0 + z_2 - a)^2 + {z_4}^2 = (a - x_2 + z_6)^2 \end{cases} \qquad (7)$$

Performing the appropriate calculations and taking into account the conditions of the system (1), we arrive at the following system:

$$z_1^2 + 2x_1^0(z_1 + z_5) - z_5^2 + z_3^2 = 0$$
$$2(x_2^0 - x_1^0)(z_2 - z_1) + z_3^2 - 2z_4z_3 + z_4^2 = 0$$
$$2z_2(x_2^0 - a) - 2z_6(a - x_2^0) + z_4^2 + z_2^2 - z_6^2 = 0$$

Using the method of Newton's polynomials, we obtain the following abbreviated system for system (7).

$$2x_1^0(z_1 + z_5) + z_3^2 = 0$$
$$2(x_2^0 - x_1^0)(z_2 - z_1) + 2z_4z_3 = 0$$
$$2z_2(x_2^0 - a) - 2z_6(a - x_2^0) + z_4^2 = 0$$

By removing this system:

$$z_1 = z_5 + \frac{z_3^2}{2x_2^0}; \qquad z_3 = \frac{(x_2^0 - x_1^0)(z_2 - z_1)}{z_4}; \qquad z_2 = \frac{z_4^2}{2(a - x_2^0)} - z_6;$$

Putting the found values of z_i $(i = \overline{1.6})$ into system (6) we get the asymptotic solution for system (5):

$$\begin{cases} x_1 = \frac{2x_1^0(x_1^0 + z_5) - z_3^2}{2x_1^0} + \dots \\ x_2 = \frac{(a - x_2^0)(x_2^0 - z_6) + z_4^2}{2(a - x_2^0)} + \cdots \\ y_1 = \frac{(x_2^0 - x_1^0)(z_2 - z_1)}{z_4} + \cdots \\ y_2 = z_4 + \cdots \\ L_1 = x_1^0 + z_5 + \cdots \\ L_2 = x_2^0 + z_6 + \cdots \end{cases} \qquad (8)$$

From the solutions (8) found for the system (5) it can be seen (diagram 3) that the mechanism can move in different directions from such a special state, and by knowing them, it is possible to eliminate the special states of the mechanism in the necessary cases, that is, the mechanism it will be possible to ensure that it does not fall into special cases by changing its parameters.

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