Comparative Analysis of Euclid, Lobachevsky and Rieman Geometries

Hakimova D. A., Shamsiyeva O'. N., To'rayev A. T.
Assistant, Tashkent State University of Transport

A B S T R A C T
This article is devoted to solving some geometric problems in Lobachevsky geometry, in particular, finding the distance between two points in the Lobachevsky plane, its parametric representation. In addition, Lobachevsky's and Euclid's geometries were comparatively analyzed, including: different geometric properties between them were studied.

A R T I C L E I N F O
Article history:
Received 11 Mar 2024
Received in revised form
10 Apr 2024
Accepted 07 May 2024

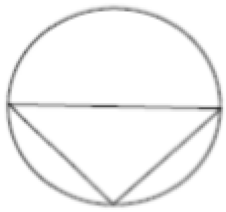
Keywords: Hippocratic problem, Euclidean, Lobachevsky and Riemannian geometries.

The Hippocratic question:

1. Leaf drawn in diameter and radius equal to $\sqrt{2}$ In this case, the surface of the leaf is equal to the surface of an equilateral right-angled triangle ACB, whose diameter D is the hypotenuse, i.e:

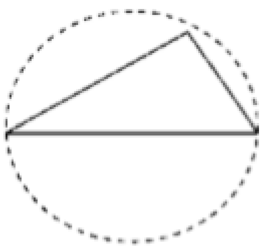
$$S_{ADByaproqcha} = S_{ACB}$$

ACB – right triangle.



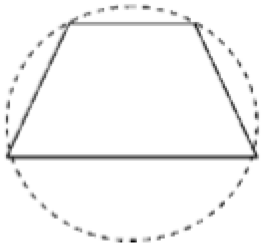
2. Circles C are made by making the sides of the triangle the diameters. In that case, the sum of the surfaces of the leaves attached to the legs is equal to the surface of the triangle ACB, i.e:

$$S_{AEB} + S_{BCF} = S_{ABC}$$



3. We make a circle drawn outside of a trapezoid with sides 1, 1, 1, $\sqrt{3}$ and $\sqrt{3}$ sides, making a segment similar to the other 3 segments. The surface of the resulting leaf is equal to the surface of the trapezoid, that is:

$$S_{ADCByaproqcha} = S_{ABCDtrapetsiya}$$



In this, Hippocrates is based on the theorem that "The ratio of the surfaces of similar segments is proportional to the square of the ratio of the diameters on which they are drawn." The answer to the question of the number of such leaves remains open. In 1840, the German mathematician Clausen found 2 more leaves. In the 20th century, Soviet mathematicians Chebotarev and Dorodnov found a complete answer to this unsolved problem, that is, if the values of the angles of the outer and inner arcs of the leaflets are reciprocal, then the problem has a solution, otherwise it does not. Accordingly, it is $\frac{2}{1}, \frac{3}{1}, \frac{3}{2}, \frac{5}{1}, \frac{5}{3}$ and the other leaves are not squared.

The very setting of the problem means that we cannot solve it with a ruler and a circle.

As a result of abstracting in solving concrete problems, solving problems of the same type, the diversity and independence of mathematics began to be revealed. These facts made it necessary to systematize mathematical knowledge and explain its foundations in a logical sequence. Philosophical worldviews of Aristotle and the achievements of the science of logic played a big role in successfully solving this task. By this time, the main forms of thinking were formed, systematized and scientifically produced, and the main principles of building a deductive science were put forward. According to this principle, a logically complex science should be built on the basis of a system of axioms. And mathematics was just such a science.

After that, mathematics began to be created on the basis of the deductive method in the form of "Fundamentals". Let's get acquainted with the most famous work of them. Euclid himself probably set a goal to write a book based on the Aristotelian principle, and as a result, an encyclopedia of mathematical knowledge will be created.

Basics consists of 13 books. Each of these has a sequence of theorems.

A definition is a sentence with the help of which the author explains mathematical concepts. For example: "a point is such that it has no part" or "a cube is such a body that it is bounded by six equal squares".

An axiom is a sentence with the help of which the author introduces equality and inequality of quantities. There are 5 axioms in total, and these are Eudoxus' system of axioms:

1. $a = b,\ b = c \Rightarrow a = c$;

2. $a = b,\ c \Rightarrow a + c = b + c$;

3. $a = b,\ c \Rightarrow a - c = b - c$;

4. $a = b \Rightarrow b = a$;

5. Bigger than the whole part.

Pastulat is a sentence with the help of which geometric constructions are confirmed and algorithmic operations are based. There are five total postulates:

1. a straight line can be drawn through any two points.

2. A straight line segment can be continued indefinitely.

3. a circle can be drawn from any center with any radius.

4. all right angles are equal.

5. If two straight lines lying in the same plane are intersected by a third straight line, and the sum of the internal one-sided angles is less than 180°, then the straight lines intersect on this side.

Now let's get acquainted with the content of "Beginnings". Books I - VI are devoted to planametry.

Books VII - IX are devoted to arithmetic.

Book X is devoted to numbers incommensurable to biquadratic irrationalities.

Books XI-XII are devoted to stereometry.

A.N. Kolmogorov "In order to master all the various structures studied in modern mathematics, it is necessary to know the axiomatic method that systematically helps the possibilities of development that are realized depending on the appearance and change of the initial axioms of a theory adapted to it" (Historical mathematician, Russia, Nauka, M. 1968, 264 p.)

Geometry viewed based on axioms that do not involve the axiom of parallelism is called "absolute" geometry by a special name.

The second category of theorems includes theorems that cannot be proven without the axiom of parallelism. These theorems of Euclidean geometry do not hold in Lobachevsky and Riemannian geometry. However, theorems of the second category of Euclidean geometry correspond to theorems derived from its axiom of parallelism in Lobachevsky geometry. And in Riemannian geometry, this is not always the case, because in this geometry there is no concept of parallelism at all.

Expression of some theorems in Euclidean, Lobachevsky and Riemann geometries

In Euclidean geometry

The sum of the interior angles of a triangle is 2d.

The exterior angle of a triangle is equal to the sum of its two non-adjacent interior angles.

The side opposite the 30° angle of a right triangle is equal to half the hypotenuse.

Angles with correspondingly parallel sides are congruent if they are both acute or both obtuse.

The sum of the interior angles of a rectangle is 4d.

The square of the hypotenuse of a right triangle is equal to the sum of the squares of its legs.

In Lobachevsky geometry

The sum of the interior angles of a triangle is less than 2d.

An exterior angle of a triangle is greater than the sum of its two non-adjacent interior angles.

The leg opposite the 30° angle of a right triangle is greater than half of the hypotenuse.

Angles with correspondingly parallel sides are unequal.

The sum of the interior angles of a quadrilateral is less than 4d.

The square of the hypotenuse of a right triangle is greater than the sum of the squares of the legs

In Riemannian geometry

The sum of the interior angles of a triangle is greater than 2d

An exterior angle of a triangle is less than the sum of its two non-adjacent interior angles.

The leg opposite the 30° angle of a right triangle is less than half of the hypotenuse.

Parallel straight lines do not exist.

The sum of the interior angles of a rectangle is greater than 4d.

The square of the hypotenuse of a right triangle is less than the sum of the squares of the legs.

Summary

This article is devoted to solving examples of how to make. The article is devoted to the comparative analysis of Euclidean and Lobachevsky geometries. It is shown that theorems of the second category of Euclid's geometry correspond to theorems derived from its axiom of parallelism in Lobachevsky's geometry. The article presents some theorems in Euclidean, Lobachevsky and Riemann geometries.

List of used literature:

1. К.Л.Самаров, В.М.Уроев. Модель Пуанкаре, kvant.mccme.ru.
2. А.Ширшов, Модель Кели-Клейна. Геометрии Лобачевского, «Квант», № 3, 1976.
3. Н.В.Эфимов. Высшая геометрия М. Наука. Главная редакция физ.-мат. лит.- 1978. -576 с.
4. А.В.Погорелов. Геометрии. М., Наука, 1968.
5. Широков П.А. Краткий очерк основ геометрии Лобачевского. М. Наука. – 1983. – 80 с.
6. Розенфельд Б.А. Неевклидовы пространства. М. Наука. -1969. -548 с.