European Journal of Business Startups and Open Society

Vol. 3 No. 2 (February - 2023): EJBSOS

http://innovatus.es/index.php/ejbsos

Cauchy-Buniakovsky Inequality and its Applications (the Smallest Value of the Sum, the Largest Value of the Product)

Ruziyev Jamshid Xudoyberdiyevich

Academic Lyceum of Tashkent State University of Economics, Teacher of Mathematics

Abstract: In this article, the Cauchy-Buniakovsky inequality and its applications (the smallest value of the sum. the largest value of the product) are described in detail and comprehensively.

Key words: theorem, proof, equation, inequality, geometry, etc

The bright star of mathematics, the great French scientist Augustine Louis Cauchy was born in 1789 in a noble family. In 1807, he graduated from the famous polytechnic school in Paris, which prepares highly qualified engineers. Since 1810, he worked as an engineer in Cherbourg. Cauchy was involved in various fields: theory of elasticity, optics, celestial mechanics, differential equations, geometry, algebra and number theory. The basis of Cauchy's interests was mathematical analysis. He is one of the founders of mathematical analysis and the theory of functions with complex variables. In 1816, Cauchy was accepted as a member of the Paris Academy of Sciences and began working as a professor at the Polytechnic School. Here he gave his famous lectures on mathematical analysis. These expositions were later published in the form of three books: "Course of Analysis" (1821), "Summary of Lectures on the Calculus of Infinitesimals" (1823), "Lectures on Applications of Analysis to Geometry" (1826-1828). -y.) There are many theorems and terms associated with Cauchy's name in higher mathematics, for example:

- Cauchy's uniqueness theorem for convex polynomials,
- Cauchy index for polynomials,
- Cauchy's inequality for arithmetic mean and geometric mean of non-negative numbers,
- Bolsano-Cauchy theorem for continuous functions,
- Cauchy-type integral,
- Cauchy's formula for the gamma function,
- Cauchy-Bunyokovskii inequality,
- the Bine-Cauchy theorem in the theory of determinants,
- Cauchy's theorem in the theory of groups,
- koshi sign in number lines,
- Cauchy-Adamard formula,
- cauchy problem for differential equations,
- Cauchy-Riemann condition for complex variable functions,
- Finding a particular solution of a non-homogeneous linear differential equation Cauchy's method occupies an important place in mathematics.

German mathematician Felix Klein praised Cauchy saying, "According to his outstanding achievements in all fields of mathematics, he can be placed almost next to Gauss." Russian mathematician, academician A.D. Aleksandrov said that "Cauchy's thinking in proving the theorem of uniqueness for convex polynomials is one of the most amazing thoughts in geometry." Augustine Louis Cauchy died in 1857. During his life, he wrote 789 scientific works, these works are embodied in 25 large volumes. Today, Augustine Louis Cauchy's methods have become classical methods. In other words, the geometric mean of non-negative numbers does not exceed their arithmetic mean, and equality is fulfilled only when these numbers are equal to each other.

$$a_1 + a_2 + ...a_n \geq 0$$

In order to show the correctness of the inequality, it is very difficult in some cases to determine whether the difference of both its parts is positive or negative, that is, to try to prove it using the same definition as in example 1. Therefore, properties of inequalities are used in proving inequalities.

The Cauchy-Bunyakovsky-Schwarz inequality, or for short, the (CBS)− inequality, plays an important role in different branches of Modern Mathematics including Hilbert Spaces Theory, Probability & Statistics, Classical Real and Complex Analysis, Numerical Analysis, Qualitative Theory of Differential Equations and their applications. The main purpose of this survey is to identify and highlight the discrete inequalities that are connected with (CBS)− inequality and provide refinements and reverse results as well as to study some functional properties of certain mappings that can be naturally associated with this inequality such as superadditivity, supermultiplicity, the strong versions of these and the corresponding monotonicity properties. Many companions and related results both for real and complex numbers are also presented. It was one of the main aims of the survey to provide complete proofs for the results considered. We also note that in most cases only the original references are mentioned. Each section concludes with a list of the references utilized and thus may be read independently. Being self contained, the survey may be used by both postgraduate students and researchers interested in Theory of Inequalities & Applications as well as by Mathematicians and other Scientists dealing with numerical computations, bounds and estimates where the (CBS)− inequality may be used as a powerful tool. The author intends to continue this survey with another one devoted to the functional and integral versions of the (CBS)− inequality. The corresponding results holding in inner-product and normed spaces will be considered as well. (CBS) −Inequality for Real Numbers. The following inequality is known in the literature as Cauchy’s or Cauchy-Schwarz’s or Cauchy-Bunyakovsky-Schwarz’s inequality. For simplicity, we shall refer to it throughout this work as the (CBS) −inequality.

Equality holds if and only if the sequences a and b are proportional. This inequality has many applications in non-Euclidean geometry, in the theory of functional equations, and in operators theory. Equality holds if and only if sequences a and b are proportional. In the special case p = q = 2, we deduce the classical Aczél inequality. In [18], we found an approach of some bounds for several statistical indicators with the Aczél inequality, and in [19], we found a proof of the Aczél inequality given with tools of the Lorentz–Finsler geometry. Motivated by the above results, in Section 2, we study a new refinement of the C-B-S inequality for the Euclidean space and several inequalities for two bounded linear operators on the Hilbert space H, where we mention Bohr’s inequality and Bergström’s inequality Symmetry 2021, 13, 305 3 of 14 for operators. We also show an inequality of the Cauchy–Bunyakovsky–Schwarz type for bounded linear operators, by the technique of the monotony of a sequence. Finally, we prove a refinement of the Aczél inequality for bounded linear operators on the Hilbert space H. In Section 3, we present some identities for real numbers obtained from some identities for Hermitian operators. This work is important because it extends a series of inequalities for real numbers to inequalities that are true for different classes of operators. This development is not easy in most cases. We also obtain new inequalities between operators, which by choosing a particular case, can generate new inequalities for real numbers and for matrices.

References:

1. Sh.A.Alimov,A.R.Xalmuxamedov,M.A.Mirzaxmedov.Algebra.-4-nashri. Toshkent:"O'qituvchi" NMIU,2019.
2. Nasritdinov G'.N. Mirzaahmedov M.A., Usmonov F.R., Aripova Sh.R. Algebra. –T.: "G'afurG'ulom nomidagi nashryot-matbaa ijodiy uyi", 2012.
3. Alixanov S. Matematika o'qitish metodikasi. – T.: "O`qituvchi", 2008.
4. "Algebra va matematika analiz asoslari" 2006.
5. A. A'zamov.B.Haydarov.E.Sariqov.Geometriya.-Tuzatilgan va to'ldirilgan uchinchi nashr.-Toshkent.: "Yangiyo'l poligraf servis", 2017.